

Axial Stringy System of the Kerr Spinning Particle *

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Abstract

The structure of classical spinning particle based on the Kerr-Newman black hole (BH) solution is investigated. For large angular momentum, $|a| \gg m$, the BH horizons disappear exposing a naked ringlike source which is a circular relativistic string. It was shown recently that electromagnetic excitations of this string lead to the appearance of an extra axial stringy system which consists of two half-infinite strings of opposite chirality. In this paper we consider the relation of this stringy system to the Dirac equation.

We show that the axial strings are the Witten superconducting strings and describe their structure by the Higgs field model where the Higgs condensate is used to regularize axial singularity. We argue that this axial stringy system may play the role of a classical carrier of the wave function.

1 Introduction

The Kerr rotating black hole solution displays some remarkable relations to spinning particles [1, 2, 3, 4, 5, 6, 7, 8, 9]. For the parameters of elementary

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particles, $|a| \gg m$, and black-hole horizons disappear. This changes drastically the usual black hole image since there appear the rotating source in the form of a closed singular ring of the Compton radius $a = J/m$.¹ In this work we continue investigation of the classical spinning particle based on the Kerr-Newman black hole (BH) solution.

One of the first papers of this series was the work [3] published 30 years ago. In this paper the model of the Kerr spinning particle - “microgeon” was suggested which was based on the Wheeler’s idea of the “mass without mass”. The Kerr ring was considered as a gravitational waveguide for the traveling electromagnetic (and fermionic) wave excitations.

That time the central Russian theoretical seminars in Moscow were held by Dmitrii Dmitrievich Ivanenko at Moscow State University. At this seminar I learnt what is the Kerr solution and strings. My first talk on the geon model, about 1972, was met by Dmitrii Dmitrievich with some degree of skepticism. However later, after discussions, his opinion was changed, and in 1975 we published a common letter on “Gravitational strings in the models of elementary particles” [4] which contained the conjecture that the Kerr singular ring was the string. By preparing of this paper Dmitrii Dmitrievich said “It may be very important if it is only true in a small degree.”

The assumption that the Kerr singular ring is the string was based on some evidences of Refs. [6, 10, 11]. However, the attempts to show it rigorous ran into obstacles which were related with the very specific motion of the Kerr ring - the lightlike sliding along itself. It could be described as a string containing lightlike modes of only one direction. However, the system of the bosonic string equations does not admit such solutions. Only thirty years later this problem was resolved. In the recent paper [12], it was shown that the Kerr ring satisfies all the stringy equations representing a string with an orientifold world sheet.

In this paper we discuss in details the second stringy structure of the Kerr spinning particle which was obtained only recently [13].² We show that the aligned electromagnetic excitations of the Kerr circular string unavoidably lead to the appearance of the axial half-infinite strings which are similar to the Dirac monopole string and topologically coupled to the Kerr ring. The class

¹Here J is angular momentum and m is mass. We use the units $c = \hbar = G = 1$, and signature $(-+++)$.

²It should be noted, that in our old common paper with Dmitrii Dmitrievich [4] the existence of this second stringy system in the Kerr spinning particle was mentioned too.

of the aligned e.m. solutions turns out to be very restricted: all the solutions can be numbered by the integer index $n = 0, \pm 1, \pm 2, \dots$, and, for the exclusion $n = 0$, they lead to the appearance of axial half-infinite singularities. We show that these axial strings carry the chiral traveling waves induced by the e.m. excitations of the Kerr circular string.

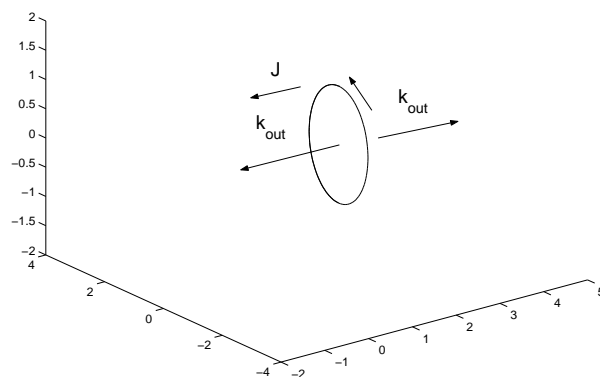


Figure 1: Stringy skeleton of the Kerr spinning particle. Circular D-string and the directed outwards two axial half-infinite chiral D-strings.

Therefore, the frame of the Kerr spinning particle turns out to be consisting of two topologically coupled stringy systems. The appearance of the axial half-infinite strings looks strange at first sight. However, we obtain that it can be a new and very important element of the structure of spinning particles, since it has the relation to the Dirac equation. We show that for the moving particle the excitations of the chiral strings are modulated by de Broglie periodicity, and therefore, the axial stringy system turns out to be a carrier of the wave function.

In the zone which is close to the Kerr string, our treatment is based on the Kerr-Schild formalism [14] and previous paper [15] where the real and complex structures of the Kerr geometry were considered. For the reader convenience we describe briefly the necessary details of these structures. Meanwhile, in the far zone, structure of this string is described by the very simple class of pp-wave solutions [16, 17]. The resulting stringy frame turns out to be very simple and easy for description. We obtain that these strings belong

to the class of the chiral superconducting strings which have recently paid considerable attention in astrophysics [18, 19, 20]. In fact these chiral strings turns out to be the Witten's superconducting strings [21, 22]. Note, that the similar chiral strings are also very popular in different models of the high dimensional superstring theory, forming the fundamental strings [23], chiral null systems [24], multiply wound strings [25] and supertubes [26].

During the treatment we meet some singular structures and divergences which have to be regularized. In particular, the mass of the infinite axial stringy system is divergent and we perform its renormalization, which shows that tension of the axial stringy system tends to zero for a free particle, but can take a finite value for a bounded system. The field singularities of the stringy pp-waves have to be also regularized, which is realized by introduction of a superconducting source for these strings. It is remarkable that the considered procedures of regularization, having the clear physical meaning in the stringy Kerr geometry, resemble the regularizations used in QED [27, 28].

2 The Kerr geometry and the Kerr circular string.

We use the Kerr-Schild approach to the Kerr geometry [14], which is based on the Kerr-Schild form of the metric

$$g_{\mu\nu} = \eta_{\mu\nu} + 2hk_\mu k_\nu, \quad (1)$$

where $\eta_{\mu\nu}$ is the metric of auxiliary Minkowski space-time, $h = \frac{mr - e^2/2}{r^2 + a^2 \cos^2 \theta}$, and k_μ is a twisting null field, which is tangent to the Kerr principal null congruence (PNC) and is determined by the form ³

$$k_\mu dx^\mu = dt + \frac{z}{r} dz + \frac{r}{r^2 + a^2} (x dx + y dy) - \frac{a}{r^2 + a^2} (x dy - y dx). \quad (2)$$

The form of the Kerr PNC is shown in Fig. 2. It follows from Eq.(1) that the field k^μ is null with respect to $\eta_{\mu\nu}$ as well as with respect to the full metric $g_{\mu\nu}$,

$$k^\mu k_\mu = k^\mu k^\nu g_{\mu\nu} = k^\mu k^\nu \eta_{\mu\nu}. \quad (3)$$

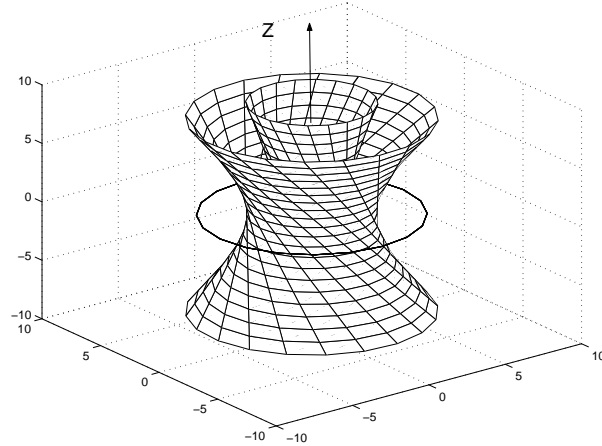


Figure 2: The Kerr singular ring and 3D section of the Kerr principal null congruence. Singular ring is a branch line of space, and PNC propagates from the “negative” sheet of the Kerr space to the “positive” one, covering the space-time twice.

The metric is singular at the ring $r = \cos \theta = 0$, which is the focal region of the oblate spheroidal coordinate system r, θ, ϕ .

The Kerr singular ring is the branch line of the Kerr space on two folds: positive sheet ($r > 0$) and ‘negative’ one ($r < 0$). Since for $|a| \gg m$ the horizons disappear, there appears the problem of the source of the Kerr solution with the alternative: either to remove this twofoldedness or to give it a physical interpretation. Both approaches have received attention, and it seems that both are valid for different models. The most popular approach was connected with the truncation of the negative sheet of the Kerr space, which leads to the source in the form of a relativistically rotating disk [2] and to the class of the disklike [5] or baglike [9] models of the Kerr spinning particle.

An alternative way is to retain the negative sheet treating it as the sheet of advanced fields. In this case the source of the spinning particle turns out to be the Kerr singular ring with the electromagnetic excitations in the form of traveling waves, which generate spin and mass of the particle. A model of this

³The rays of the Kerr PNC are twistors and the Kerr PNC is determined by the Kerr theorem as a quadric in projective twistor space [15].

sort was suggested in 1974 as a model of “microgeon with spin” [3]. The Kerr singular ring was considered as a waveguide providing a circular propagation of an electromagnetic or fermionic wave excitation. Twofoldedness of the Kerr geometry admits the integer and half integer excitations with $n = 2\pi a/\lambda$ wave periods on the Kerr ring of radius a , which turns out to be consistent with the corresponding values of the Kerr parameters $m = J/a$.

The lightlike structure of the Kerr ring worldsheet is seen from the analysis of the Kerr null congruence near the ring. The lightlike rays of the Kerr PNC are tangent to the ring.

It was recognized long ago [4] that the Kerr singular ring can be considered in the Kerr spinning particle as a string with traveling waves. One of the most convincing evidences obtained by the analysis of the axidilaton generalization of the Kerr solution (given by Sen [30]) near the Kerr singular ring was given in [6]. It was shown that the fields near the Kerr ring are very similar to the field around a heterotic string.

The analysis given in [12] showed that the worldsheet of the Kerr ring satisfies the bosonic string equations and constraints; however, there appear problems with boundary conditions, and these difficulties can be removed by the formation of the worldsheet orientifold.

The interval of an open string $\sigma \in [0, \pi]$ has to be extended to $[0, 2\pi]$, setting

$$X_R(\sigma + \pi) = X_L(\sigma), \quad X_L(\sigma + \pi) = X_R(\sigma). \quad (4)$$

By such an extension, there appear the both types of modes, “right” and “left” since the “left” modes play the role of “right” ones on the extended piece of interval. If the extension is completed by the changing of orientation on the extended piece, $\sigma' = \pi - \sigma$, with a subsequent identification of σ and σ' , then one obtains the closed string on the interval $[0, 2\pi]$ which is folded and takes the form of the initial open string.

Formally, the worldsheet orientifold represents a doubling of the worldsheet with the orientation reversal on the second sheet. The fundamental domain $[0, \pi]$ is extended to $\Sigma = [0, 2\pi]$ with formation of folds at the ends of the interval $[0, \pi]$.

3 Aligned e.m. solutions on the Kerr-Schild background

To realize the idea of the Kerr spinning particle as a “microgeon” we have to consider the electromagnetic excitations of the Kerr string which are described on the Kerr background by the aligned to the Kerr PNC solutions. Such e.m. solutions can be treated as candidates for the self-consistent Einstein-Maxwell solutions.

The treatment on this section is based on the Kerr-Schild formalism, and the readers which are not aware of this formalism can omit this part by first reading going to the physical consequences of these solutions.

The aligned field equations for the Einstein-Maxwell system in the Kerr-Schild class were obtained in [14]. Electromagnetic field is given by tetrad components of self-dual tensor

$$\mathcal{F}_{12} = AZ^2 \quad (5)$$

$$\mathcal{F}_{31} = \gamma Z - (AZ)_{,1} \quad (6)$$

The equations for electromagnetic field are

$$A_{,2} - 2Z^{-1}\bar{Z}Y_{,3}A = 0, \quad (7)$$

$$\mathcal{D}A + \bar{Z}^{-1}\gamma_{,2} - Z^{-1}Y_{,3}\gamma = 0. \quad (8)$$

Gravitational field equations yield

$$M_{,2} - 3Z^{-1}\bar{Z}Y_{,3}M = A\bar{\gamma}\bar{Z}, \quad (9)$$

$$\mathcal{D}M = \frac{1}{2}\gamma\bar{\gamma}, \quad (10)$$

where

$$\mathcal{D} = \partial_3 - Z^{-1}Y_{,3}\partial_1 - \bar{Z}^{-1}\bar{Y}_{,3}\partial_2. \quad (11)$$

Solutions of this system were given in [14] only for stationary case for $\gamma = 0$. Here we consider the oscillating electromagnetic solutions which corresponds to the case $\gamma \neq 0$.

For the sake of simplicity we have to consider the gravitational Kerr-Schild field as stationary, although in the resulting e.m. solutions the axial

symmetry is broken, which has to lead to oscillating backgrounds if the back reaction is taken into account.

The recent progress in the obtaining the nonstationary solutions of the Kerr-Schild class is connected with introduction of a complex retarded time parameter $\tau = t_0 + i\sigma = \tau|_L$ which is determined as a result of the intersection of the left (L) null plane and the complex world line [15]. The left null planes are the left generators of the complex null cones and play a role of the null cones in the complex retarded-time construction. The τ parameter satisfies to the relations

$$(\tau)_{,2} = (\tau)_{,4} = 0 . \quad (12)$$

It allows one to represent the equation (7) in the form

$$(AP^2)_{,2} = 0 , \quad (13)$$

and to get the following general solution

$$A = \psi(Y, \tau)/P^2 \quad (14)$$

which has the form obtained in [14]. The only difference is in the extra dependence of the function ψ from the retarded-time parameter τ .

It was shown in [15] that action of operator \mathcal{D} on the variables Y, \bar{Y} and ρ is following

$$\mathcal{D}Y = \mathcal{D}\bar{Y} = 0, \quad \mathcal{D}\rho = 1 , \quad (15)$$

and therefore $\mathcal{D}\rho = \partial\rho/\partial t_0 \mathcal{D}t_0 = P\mathcal{D}t_0 = 1$, that yields

$$\mathcal{D}t_0 = P^{-1}. \quad (16)$$

As a result the equation (8) takes the form

$$\dot{A} = -(\gamma P)_{,\bar{Y}} , \quad (17)$$

where $(\dot{}) \equiv \partial_{t_0}$.

For considered here stationary background $P = 2^{-1/2}(1 + Y\bar{Y})$, and $\dot{P} = 0$. The coordinates Y , and τ are independent from \bar{Y} , which allows us to integrate Eq. (17) and we obtain the following general solution

$$\gamma = -P^{-1} \int \dot{A} d\bar{Y} = -P^{-1} \dot{\psi}(Y, \tau) \int P^{-2} d\bar{Y} = \frac{2^{1/2} \dot{\psi}}{P^2 Y} + \phi(Y, \tau)/P, \quad (18)$$

where ϕ is an arbitrary analytic function of Y and τ .

The term γ in $\mathcal{F}_{31} = \gamma Z - (AZ)_{,1}$ describes a part of the null electromagnetic radiation which falls off asymptotically as $1/r$ and propagates along the Kerr principal null congruence e^3 . As it was discussed in [15, 12] it describes a loss of mass by radiation with the stress-energy tensor $\kappa T_{\mu\nu}^{(\gamma)} = \frac{1}{2}\gamma\bar{\gamma}e_\mu^3e_\nu^3$ and has to lead to an infrared divergence. However, the Kerr twofoldedness and the structure of the Kerr principal null congruence show us that the loss of mass on the positive sheet of metric is really compensated by an opposite process on the “negative” sheet of the Kerr space where is an in-flow of the radiation. In the microgeon model [15, 12, 13], this field acquires interpretation of the vacuum zero point field $T_{\mu\nu}^{(\gamma)} = \langle 0|T_{\mu\nu}|0 \rangle$. Similar to the treatment of the zero point field in the Casimir effect one has to regularize stress energy tensor by the subtraction

$$T_{\mu\nu}^{(reg)} = T_{\mu\nu} - \langle 0|T_{\mu\nu}|0 \rangle, \quad (19)$$

under the condition $T^{(\gamma)\mu\nu}_{,\mu} = 0$ which is satisfied for the γ term.

Let's now consider in details the second term in (6):

$$(AZ)_{,1} = (Z/P)^2(\psi_{,Y} - 2\psi P_Y) + (Z/P^2)\dot{\psi}\tau_{,1} + AZ_{,1}. \quad (20)$$

For stationary case we have relations $Z_{,1} = 2ia\bar{Y}(Z/P)^3$ and $\tau_{,1} = -2ia\bar{Y}Z/P^2$. This yields

$$(AZ)_{,1} = (Z/P)^2(\psi_{,Y} - 2ia\dot{\psi}\bar{Y}/P^2 - 2\psi P_Y/P) + A2ia\bar{Y}(Z/P)^3. \quad (21)$$

Since $Z/P = 1/(r + ia \cos \theta)$, this expression contains the terms which are singular at the Kerr ring and fall off like r^{-2} and r^{-3} . However, it contains also the factors which depend on coordinate $Y = e^{i\phi} \tan \frac{\theta}{2}$ and can be singular at the z-axis.

These singular factors can be selected in the full expression for the aligned e.m. fields and as a result there appear two half-infinite lines of singularity, z^+ and z^- , which correspond to $\theta = 0$ and $\theta = \pi$ and coincide with corresponding axial lightlike rays of the Kerr principal null congruence. On the “positive” sheet of the Kerr background these two half-rays are directed outward. However, one can see that they are going from the “negative” sheet and appear on the “positive” sheet passing through the Kerr ring (see Fig. 2).

The general solution for the aligned electromagnetic fields has the form

$$\mathcal{F} = \mathcal{F}_{31} e^3 \wedge e^1 + \mathcal{F}_{12} (e^1 \wedge e^2 + e^3 \wedge e^4). \quad (22)$$

In the null Cartesian coordinates the Kerr-Schild null tetrad has the form ⁴

$$\begin{aligned} e^1 &= d\zeta - Y dv, & e^2 &= d\bar{\zeta} - \bar{Y} dv, \\ e^3 &= du + \bar{Y} d\zeta + Y d\bar{\zeta} - Y\bar{Y} dv, \\ e^4 &= dv + h e^3. \end{aligned} \quad (24)$$

Evaluating the basis two-forms in the Cartesian coordinates we obtain

$$e^1 \wedge e^2 + e^3 \wedge e^4 = d\zeta \wedge d\bar{\zeta} + du \wedge dv + Y d\bar{\zeta} \wedge dv, \quad (25)$$

and

$$e^3 \wedge e^1 = Y d\bar{\zeta} \wedge d\zeta + du \wedge dz - Y du \wedge dv - Y^2 d\bar{\zeta} \wedge dv. \quad (26)$$

4 Axial stringy system

The obtained general solution for the aligned electromagnetic fields (22) contains the factors which depend on coordinate $Y = e^{i\phi} \tan \frac{\theta}{2}$ and can be singular at the z-axis.

We will now be interested in the wave terms and omit the terms describing the longitudinal components and the field γ .

The wave terms are proportional to the following basis two-forms

$$e^3 \wedge e^1|_{wave} = du \wedge d\zeta + Y^2 dv \wedge d\bar{\zeta}$$

and

$$(e^1 \wedge e^2 + e^3 \wedge e^4)|_{wave} = d\zeta \wedge d\bar{\zeta}.$$

⁴In the paper [14] treatment is given in terms of the “in” - going congruence e^3 (advanced fields). Here we need to use the “out”-going congruence. The simplest way to do it retaining the basic relations of the paper [14] is to replace $t \rightarrow -t$ in the definition of the null Cartesian coordinates. Therefore, we use here the notations

$$\begin{aligned} 2^{\frac{1}{2}}\zeta &= x + iy, & 2^{\frac{1}{2}}\bar{\zeta} &= x - iy, \\ 2^{\frac{1}{2}}u &= z - t, & 2^{\frac{1}{2}}v &= z + t. \end{aligned} \quad (23)$$

Near the positive half-axis z^+ , we have $Y \rightarrow 0$ and near the negative half-axis z^- , we have $Y \rightarrow \infty$.

Therefore, with the exclusion of the γ term, the wave terms of the e.m. field (22) have the form

$$\mathcal{F}|_{\text{wave}} = f_R d\zeta \wedge du + f_L d\bar{\zeta} \wedge dv, \quad (27)$$

where the factor

$$f_R = (AZ)_{,1} \quad (28)$$

describes the “right” waves propagating along the z^+ half-axis, and the factor

$$f_L = 2Y\psi(Z/P)^2 + Y^2(AZ)_{,1} \quad (29)$$

describes the “left” waves propagating along the z^- half-axis, and some of them are singular at z axis.

Besides, since $Z/P = (r + ia \cos \theta)^{-1}$, all the terms are also singular at the Kerr ring $r = \cos \theta = 0$. Therefore, the singular excitations of the Kerr ring turn out to be connected with the axial singular waves.

Let us consider the solutions describing traveling waves along the Kerr ring

$$\psi_n(Y, \tau) = qY^n \exp i\omega_n \tau \equiv q \left(\tan \frac{\theta}{2} \right)^n \exp i(n\phi + \omega_n \tau). \quad (30)$$

Near the Kerr ring one has $\psi = \exp i(n\phi + \omega_n t)$, and $|n|$ corresponds to the number of the wave lengths along the Kerr ring. The parameter n has to be integer for the smooth and single-valued solutions, however, as we shall see bellow, the half-integer n can be interesting too.

Meanwhile, by $Y \rightarrow 0$ one approaches to the positive z -axis where the solutions may be singular too. Similar, by $Y \rightarrow \infty$ one approaches to the negative z -axis, and some of the solutions turns out to be singular there.

When considering asymptotical properties of these singularities by $r \rightarrow \infty$, we have $z = r \cos \theta$, and for the distance ρ from the z^+ axis we have the expression $\rho = z \tan \theta \simeq 2r|Y|$ by $Y \rightarrow 0$. Therefore, for the asymptotical region near the z^+ axis we have to put $Y = e^{i\phi} \tan \frac{\theta}{2} \simeq e^{i\phi} \frac{\rho}{2r}$, and $|Y| \rightarrow 0$, while for the asymptotical region near the z^- axis $Y = e^{i\phi} \tan \frac{\theta}{2} \simeq e^{i\phi} \frac{2r}{\rho}$, and $|Y| \rightarrow \infty$.

The parameter $\tau = t - r - ia \cos \theta$ takes near the z -axis the values

$$\tau_+ = \tau|_{z^+} = t - z - ia, \quad \tau_- = \tau|_{z^-} = t + z + ia. \quad (31)$$

It has also to be noted that for $|n| > 1$ the solutions contain the axial singularities which do not fall off asymptotically, but are increasing. Therefore, we shall restrict the treatment by the cases $|n| \leq 1$.

The leading singular wave for $n = 1$ is

$$\mathcal{F}_1^- = \frac{4qe^{i2\phi+i\omega_1\tau_-}}{\rho^2} d\bar{\zeta} \wedge dv. \quad (32)$$

This wave propagates to $z = -\infty$ and has the uniform axial singularity at z^- of order ρ^{-2} .

Meanwhile, the leading singular wave for $n = -1$ is

$$\mathcal{F}_{-1}^+ = -\frac{4qe^{-i2\phi+i\omega_{-1}\tau_+}}{\rho^2} d\zeta \wedge du, \quad (33)$$

and has the similar uniform axial singularity at z^+ which propagates to $z = +\infty$.

The waves with $n = 0$ are regular.

In what follows we will show that these singularities form the half-infinite chiral strings, in fact superconducting D-strings. There are several arguments in favor of the system containing a combination of two strings of opposite chirality, $n = \pm 1$, with $\omega_1 = -\omega_{-1} = \omega$.

First, if the solution contains only one half-infinite string, like the Dirac monopole string, it turns out to be asymmetric with respect to the z^\pm half-axis, which leads to a nonstationarity via a recoil.

Then, the symmetric stringy solutions exclude the appearance of monopole charge.

Similar to the case with the Kerr circular string, the pure chiral strings, containing modes of only one direction, cannot exist and any chiral string has to be connected to some object containing an anti-chiral part. Indeed, the pure chiral excitation depends only on one of the parameters $\tau_\pm = t \pm \sigma$, and as a result the world-sheet is degenerated in a world-line.⁵ This is seen in the models of the cosmic chiral strings where the chiral excitations are joined to some mass [18] or are sitting on some string having modes of opposite chirality [20]. In our case the partial pp-wave e.m. excitation has the same chirality as the half-infinite carrier of this excitation (the axial ray of PNC). Therefore, the combination of two $n = \pm 1$ excitations looks very

⁵This argument was suggested by G. Alekseev.

natural and leads to the appearance of a full stringy system with two half-infinite singular D-strings of opposite chirality, “left” and “right”, as it is shown at the fig.1. The world-sheet of the system formed by two straight chiral strings will be given by

$$x^\mu(t, z) = \frac{1}{2}[(t - z)k_R^\mu + (t + z)k_L^\mu], \quad (34)$$

where the lightlike vectors k^μ are constant and normalized. At the rest frame the timelike components are equal $k_R^0 = k_L^0 = 1$, and the spacelike components are oppositely directed, $k_R^a + k_L^a = 0$, $a = 1, 2, 3$. Therefore, $\dot{x}^\mu = (1, 0, 0, 0)$, and $x'^\mu = (0, k^a)$, and the Nambu-Goto string action

$$S = \alpha'^{-1} \int \int \sqrt{(\dot{x})^2 (x')^2 - (\dot{x}x')^2} dt dz \quad (35)$$

can be expressed via k_R^μ and k_L^μ .

To normalize the infinite string we have to perform a renormalization putting $\alpha'^{-1} \int (x')^2 dz = m$, which yields the usual action for the center of mass of a pointlike particle

$$S = m \int \sqrt{(\dot{x})^2} dt. \quad (36)$$

As a consequence of this renormalization we obtain that the string tension $T = \alpha'^{-1}$ is at least close to zero for a free particle. However, tension can appear in the bounded states where the axial strings may form the closed loops.

For the system of two straight D-strings in the rest one can use the gauge with $\dot{x}^0 = 1$, $\dot{x}^a = 0$, where the term $(\dot{x}x')^2$ drops out, and the action takes the form

$$S = \alpha'^{-1} \int dt \int \sqrt{p^a p_a} d\sigma, \quad (37)$$

where

$$p^a = \partial_\sigma x^a = \frac{1}{2}[x_R'^\mu(t + \sigma) - x_L'^\mu(t - \sigma)]. \quad (38)$$

However, one of the most important arguments in the favor of the above combination of two chiral strings is suggested by the relation to the Dirac equation, which must have a physical sense in the structure of the Kerr spinning particle if it pretends on a classical description of the electron.

5 Relation to the Dirac equation

It is known that in the Weyl basis the Dirac current can be represented as a sum of two lightlike components of opposite chirality⁶

$$J_\mu = e(\bar{\Psi}\gamma_\mu\Psi) = e(\chi^+\sigma_\mu\chi + \phi^+\bar{\sigma}^\mu\phi), \quad (42)$$

where

$$\Psi = \begin{pmatrix} \phi_\alpha \\ \chi^{\dot{\alpha}} \end{pmatrix}, \quad (43)$$

and

$$\bar{\Psi} = (\chi^+, \phi^+) \quad (44)$$

It allows one to conjecture that the Dirac equation may describe the Kerr axial stringy system - the lightlike currents of two half-infinite chiral strings. Each of these strings is formed from the spinors which satisfy the massless Dirac equation. The problem is to get the four-component spinor which will satisfy the massive Dirac equations

$$(\gamma^\mu \hat{P}_\mu - m)\Psi = 0, \quad \hat{P}_\mu = i\hbar\partial_\mu, \quad (45)$$

which split in this basic into two systems

$$m\phi_\alpha = i\sigma_{\alpha\dot{\alpha}}^\mu\partial_\mu\chi^{\dot{\alpha}}, \quad m\chi^{\dot{\alpha}} = i\bar{\sigma}^{\mu\dot{\alpha}\alpha}\partial_\mu\phi_\alpha, \quad (46)$$

⁶We use the spinor notations of the book [34].

$$\gamma_\mu = \begin{pmatrix} 0 & \sigma_\mu \\ \bar{\sigma}_\mu & 0 \end{pmatrix}, \quad (39)$$

where

$$\bar{\sigma}_\mu^{\dot{\alpha}\alpha} = \epsilon^{\dot{\alpha}\dot{\beta}}\epsilon^{\alpha\beta}\sigma_{\mu\beta\dot{\beta}}, \quad (40)$$

and

$$\begin{aligned} \sigma_0 = \bar{\sigma}_0 &= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \sigma_1 = -\bar{\sigma}_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \\ \sigma_2 = -\bar{\sigma}_2 &= \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_3 = -\bar{\sigma}_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \end{aligned} \quad (41)$$

Let us recall now that the used in the Kerr-Schild null tetrad function Y (24) is the projective spinor coordinate $Y = \phi_2/\phi_1$ [15]. Near the z^+ half-axis we have $Y \rightarrow 0$, and one can set in this limit

$$\phi_\alpha = \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \phi^\alpha = \epsilon^{\alpha\beta} \phi_\beta = \begin{pmatrix} 0 \\ -1 \end{pmatrix}. \quad (47)$$

This spinor describes the lightlike vector

$$k_R = d(t - z) = \bar{\phi}_{\dot{\alpha}} \bar{\sigma}_\mu^{\dot{\alpha}\alpha} dx^\mu \phi_\alpha = (1, 0) \bar{\sigma}_\mu dx^\mu \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad (48)$$

since

$$\sigma_\mu dx^\mu = \begin{pmatrix} dt + dz & dx - idy \\ dx + idy & dt - dz \end{pmatrix}, \quad \bar{\sigma}_\mu dx^\mu = \begin{pmatrix} dt - dz & -dx + idy \\ -dx - idy & dt + dz \end{pmatrix}. \quad (49)$$

Similar, near the z^- half-axis we have $Y \rightarrow \infty$, and this limit corresponds to the spinor

$$\chi_\alpha = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad \chi^\alpha = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad (50)$$

which describes the lightlike vector⁷

$$k_L = d(t + z) = \chi^\alpha \sigma_{\mu\alpha\dot{\alpha}} dx^\mu \bar{\chi}^{\dot{\alpha}} = (1, 0) \sigma_\mu dx^\mu \begin{pmatrix} 1 \\ 0 \end{pmatrix}. \quad (51)$$

The lightlike vectors k_L and k_R are the generators of the left and right chiral half-infinite strings correspondingly.

Since the spinor functions ψ and χ are fixed up to arbitrary gauge factors, one can form the four component spinor function

$$\Psi = \mathcal{M}(p_\mu x^\mu) \begin{pmatrix} a\phi_\alpha \\ b\chi^{\dot{\alpha}} \end{pmatrix}, \quad (52)$$

which may satisfy the Dirac equation.

Indeed, substitution (47), (50) and (52) into (46) leads to the equations

$$am = (p_0 - p_z)b \ln \mathcal{M}', \quad bm = (p_0 + p_z)a \ln \mathcal{M}', \quad p_x + ip_y = 0, \quad (53)$$

⁷Note, that for commuting spinors $\chi \sigma_\mu \bar{\phi} = \bar{\phi} \bar{\sigma}_\mu \chi$.

which realize the Dirac idea on splitting the relation $p_0^2 = m^2 + p_z^2$. Other necessary conditions yield $p_x = p_y = 0$. One sees that this solution corresponds to an arbitrary relativistic motion along the axial stringy system, which retains axial symmetry of this system. The function $\mathcal{M} = e^{-i\omega t + izp_z}$ oscillates with the Compton frequency which is determined by excitations of the Kerr circular string and contains a plane fronted modulation of the chiral strings by de Broglie periodicity.

One can conjecture which changes could be performed to get the Kerr anti-particle. It has to be the change of the PNC direction $k_{out} \rightarrow k_{in}$, which can be achieved by the transition to the negative sheet of the metric. It yields a natural picture of annihilation as it is shown in the fig.3. It was discussed in [32] that the size of the Kerr circular string for the massless Kerr spinning particle has to grow to infinity and disappear. As a result there retains only a single chiral string which may correspond to a massless particle.

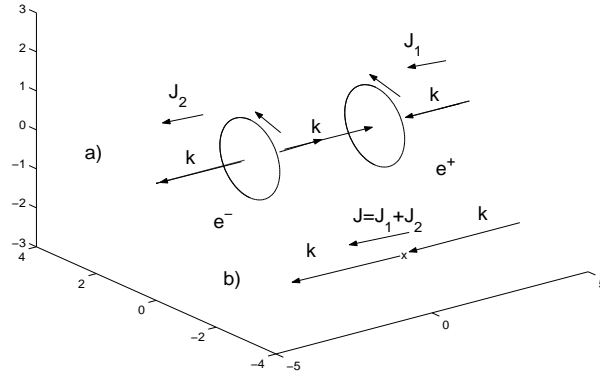


Figure 3: (a) annihilation of the Kerr particle and antiparticle and (b) formation of the lightlike particle.

To form a symmetric solution we used a combination of the $n = \pm 1$ excitations

$$\psi_n(Y, \tau)(Z/P)^2 \simeq qe^{in\phi + i\omega_n\tau} 2^{-n} \rho^n / r^{n+2} \quad (54)$$

with $\omega_1 = -\omega_{-1} = \omega$. Indeed, also the solution containing combination of three terms with $n = -1, 0, 1$ and $\omega_0 = 0$ represents especial interest since it

yields a smooth e.m. field packed along the Kerr string with one half of the wavelength and gives an electric charge to the solution.

Note, that orientifold structure of the Kerr circular string admits apparently the excitations with $n = \pm 1/2$ too, so far as the negative half-wave can be packed on the covering space turning into positive one on the second sheet of the orientifold. However, the meaning of this case is unclear yet, and it demands a special consideration.

6 Einstein-Maxwell axial pp-wave solutions

The e.m. field given by (27), (28) and (29) can be obtained from the potential

$$\mathcal{A} = -AZe^3 - \chi d\bar{Y}, \quad (55)$$

where $A = \psi/P^2$ is given by (14) and

$$\chi = \int P^{-2} \psi dY, \quad (56)$$

\bar{Y} being kept constant in this integration. The considered wave excitations have the origin from the term

$$\mathcal{A} = P^{-2} \psi_n Z e^3 = q Y^n \exp i\omega\tau P^{-2} Z e^3 \quad (57)$$

and acquire the following asymptotical z^\pm forms:

For $n = 1; z < 0$

$$\mathcal{A}^- = q Y e^{i\omega_n \tau} (r + ia \cos \theta)^{-1} e^3 / P \simeq -2q \frac{e^{i\omega_1 \tau_- + i\phi}}{\rho} dv. \quad (58)$$

For $n = -1; z > 0$

$$\mathcal{A}^+ = q Y^{-1} e^{i\omega_n \tau} (r + ia \cos \theta)^{-1} e^3 / P \simeq 2q \frac{e^{i\omega_{-1} \tau_+ - i\phi}}{\rho} du. \quad (59)$$

Each of the partial solutions represents the singular plane-fronted e.m. wave propagating along z^+ or z^- half-axis without damping. It is easy to point out the corresponding self-consistent solution of the Einstein-Maxwell field equations which belongs to the well known class of pp-waves [16, 17].

The metric has the Kerr-Schild form

$$g_{\mu\nu} = \eta_{\mu\nu} + 2hk_\mu k_\nu, \quad (60)$$

where function h determines the Ricci tensor

$$R^{\mu\nu} = -k^\mu k^\nu \square h, \quad (61)$$

$k^\mu = e^{3\mu}/P$ is the normalized principal null direction (in particular, for the z^+ axis $k^\mu dx^\mu = -2^{1/2}du$), and \square is a flat D'Alembertian

$$\square = 2\partial_\zeta \partial_{\bar{\zeta}} + 2\partial_u \partial_v. \quad (62)$$

The Maxwell equations take the form

$$\square \mathcal{A} = J = 0 \quad (63)$$

and can easily be integrated leading to the solutions

$$\mathcal{A}^+ = [\Phi^+(\zeta) + \Phi^-(\bar{\zeta})]f^+(u, v)du, \quad (64)$$

$$\mathcal{A}^- = [\Phi^+(\zeta) + \Phi^-(\bar{\zeta})]f^-(u, v)dv, \quad (65)$$

where Φ^\pm are arbitrary analytic functions, and functions f^\pm describe the arbitrary retarded and advanced waves. In our case we have the retarded-time parameter $\tau = t - r - ia \cos \theta$ which takes at the z^+ axis the values $\tau \simeq -2^{1/2}u - ia$ and at the z^- axis the values $\tau \simeq 2^{1/2}v + ia$. Therefore, we have

$$f^+ = f^+(u), \quad f^- = f^-(v). \quad (66)$$

The corresponding energy-momentum tensor will be

$$T^{\mu\nu} = \frac{1}{8\pi} |\mathcal{F}_{-1}^+|^2 k^\mu k^\nu, \quad (67)$$

where for z^+ wave $k_\mu dx^\mu = -2^{1/2}du$ and for z^- wave $k_\mu dx^\mu = 2^{1/2}dv$.

The Einstein equations $R^{\mu\nu} = -8\pi T^{\mu\nu}$ take the simple asymptotic form

$$\square h = |\mathcal{F}_{-1}^+|^2 = 16q^2 e^{-2a\omega} \rho^{-4}. \quad (68)$$

This equation can easily be integrated and yields the singular solution

$$h = 8q^2 e^{-2a\omega} \rho^{-2}. \quad (69)$$

Therefore, the wave excitations of the Kerr ring lead to the appearance of singular pp-waves which propagate outward along the z^+ and/or z^- half-axis.

These axial singularities are evidences of the axial stringy currents, which are exhibited explicitly when we regularize the singularities on the base of the Witten field model for the cosmic superconducting strings [21, 22].

The resulting excitations have the Compton wave length which is determined by the size of the Kerr circular string. However, for the moving systems the excitations of the axial stringy system are modulated by de Broglie periodicity.

7 Regularization. Superconducting strings as sources of axial pp-waves

The singular lines are often considered as strings. It is assumed that singularity is only an approximation which has to be replaced by a regular matter source in a more general field model. This point of view was considered for interpretation of the Dirac monopole singular line, and the other very well known example is the Nielsen-Olesen string model in the form of a vortex line in superconductor [33]. Both these examples are related to our case and the Higgs field model for the stringy sources is used as the most simple and adequate to these cases. Many physical stringy models as well as models of bags and domain walls are based on the different modifications of the Higgs field model. Almost all of the models leads to the picture of an extended physical object (string, bag or other) which is suited in the vacuum possessing superconducting properties. Therefore, the infinite external space surrounding these objects turns out to be superconducting. Although there is no usually the exact analytical solutions, the models turn out to be rather simple and solvable numerically. Meanwhile, the main physical discrepancy is not discussed usually: the real surrounding vacuum is not superconducting! Electromagnetic fields are freely propagating there, they are long range and do not acquire a mass from the Higgs field as it follows from these models. The physically right picture ought to be turned over: the superconducting object is to be surrounded by a real vacuum with long range electromagnetic field. Therefore, the considered usually models of extended objects describe the picture which is “dual” in some sense to the real phys-

ical situation⁸ Meanwhile, in spite of this seeming failure, the use of these dual models is striking productive. The resolution of this contradiction lies in the assumption that there are two type of superconductivity, the usual “true” one and “dual” or “false” one, and our observable “true” vacuum is not superconducting with respect to the usual electromagnetic field, $U(1)$ gauge, but possesses a “dual” or “false” superconductivity with respect to some other $\tilde{U}(1)$ gauge field which can be confined on the extended objects.

A model of this type, the $U(1) \times \tilde{U}(1)$ field model, was suggested by Witten for the cosmic superconducting strings [21, 22] and represents a doubling of the usual Abelian Higgs model.

It contains two sectors, say A and B , with two Higgs fields ϕ_A and ϕ_B , and two gauge fields A_μ and B_μ yielding two sorts of superconductivity A and B . The gauge field A_μ of the A sector is the usual electromagnetic field which is long range in outer region and acquires a mass interacting with the chiral scalar field of this sector ϕ_A . This scalar field ϕ_A can be concentrated on the extended superconducting objects and describes the superconducting vacuum state.

The sector B of the model describes a “dual” picture. The nonzero chiral field ϕ_B covers almost all our space for the exclusion of the local regions occupied by superconducting objects, or for the exclusion of the regions of localization of the field ϕ_A . Therefore, the localizations of the vacuums A and B are dual to each other. Similarity of the sectors A and B allows one to consider field ϕ_B also as a carrier of some kind of superconductivity, but this is a “false” or “dual” superconductivity which covers almost all our empty space. Therefore, our usual physical vacuum is considered as a “dual” superconductor in this model.

The corresponding Lagrangian of the Witten $U(I) \times \tilde{U}(I)$ field model is given by [21]

$$L = -(D^\mu \phi_A)(\overline{D_\mu \phi_A}) - (\tilde{D}^\mu \phi_B)(\overline{\tilde{D}_\mu \phi_B}) - \frac{1}{4} F_A^{\mu\nu} F_{A\mu\nu} - \frac{1}{4} F_B^{\mu\nu} F_{B\mu\nu} - V, \quad (70)$$

where $F_{A\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and $F_{B\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$ are field stress tensors, and the potential has the form

$$V = \lambda(\bar{\phi}_B \phi_B - \eta^2)^2 + f(\bar{\phi}_B \phi_B - \eta^2)\bar{\phi}_A \phi_A + m^2 \bar{\phi}_A \phi_A + \mu(\bar{\phi}_A \phi_A)^2. \quad (71)$$

⁸We are sorry for use the word “dual” which has to many other physical meanings.

Two Abelian gauge fields A_μ and B_μ interact separately with two complex scalar fields ϕ_B and ϕ_A so that the covariant derivative $D_\mu\phi_A = (\partial + ieA_\mu)\phi_A$ is associated with A sector, and covariant derivative $\tilde{D}_\mu\phi_B = (\partial + igB_\mu)\phi_B$ is associated with B sector. Field ϕ_B carries a $\tilde{U}(1)$ charge $\tilde{q} \neq 0$ and a $U(1)$ charge $q = 0$, and field ϕ_A carries $\tilde{U}(1)$ and $U(1)$ charges of $\tilde{q} = 0$ and $q \neq 0$, respectively.

The A and B sectors are almost independent interacting only through the potential term for scalar fields. This interaction has to provide a synchronized phase transitions from superconducting B -phase inside the bag to superconducting A -phase in the outer region. The synchronization of this transition occurs explicitly in a supersymmetric version of this model given by Morris [29]. Application of this model to the Kerr source is discussed in [9].⁹

In this section we consider in details only A sector which describes the singular e.m. excitations of the Kerr ring interacting with a superconducting stringlike source, which regularizes these singularities. For simplicity we restrict ourself by the treatment in a flat background. We put $\phi_A = \Phi e^{i\chi}$ and ignore the gravity and the fields of sector B . The necessary form of the potential V we will discuss later.

The field equations in this case take the form

$$D_\mu D^\mu \Phi e^{i\chi} + \frac{1}{2} \partial_\phi V = 0, \quad (72)$$

and

$$\square A_\mu = J_\mu = e\Phi^2(-2\chi_{,\mu} + eA_\mu). \quad (73)$$

It is known that the problems of this sort do not have explicit analytical solutions [20]. We consider the far zone near the z^+ -axis, and from (59) we have the only singular nonzero component of the massless e.m. field in the far zone $A_u = \frac{2q}{\rho} e^{i\omega\tau_+ - i\phi}$. By using (31) and the coordinates $\zeta = 2^{-1/2}\rho e^{i\phi}$ and $\bar{\zeta} = 2^{-1/2}\rho e^{-i\phi}$ we obtain

$$A_u = A_u^{(0+)} = \frac{2C}{\zeta} e^{-i2^{1/2}\omega u}, \quad (74)$$

⁹As it showed the treatment in [9], the both (false and true) vacua are to be the supersymmetric states. These vacua are separated by a D2-brane (domain wall) which has to take a tubelike form by application of this field model to the regularized axial pp-waves. The resulting structure turns out to be similar to the supersymmetric tubes considered in [26].

where

$$C = 2^{-1/2} q e^{a\omega}. \quad (75)$$

The real vector potential will be

$$A_u^{(0)} = \frac{1}{2}(A_u^{(0+)} + A_u^{(0-)}), \quad (76)$$

where

$$A_u^{(0-)} = \frac{2C}{\zeta} e^{i2^{1/2}\omega u}. \quad (77)$$

Computations turn out to be easy in this form. Index ⁽⁰⁾ we use to underline that this component is long range and massless. The physics of the process suggests us the way to solve these equations. The singular wave $A_u^{(0)}$ is to be regularized penetrating into the Higgs field $\Phi e^{i\chi}$ which is condensed on the string. Regularization can be performed by a mechanism of compensation which is similar to the Feynman and Pauli-Villars scheme of regularization. A massive short range field $A_u^{(m)}$, having just the same structure near the singularity as the field $A_u^{(0)}$, has to compensate singularity leading to a regular behavior of the sum $A_u^{(0)} + A_u^{(m)}$.

Therefore, we have to split the eq.(73) in the sum of two equations:

- one of them for the massless components $A_u^{(0\pm)}$,

$$\square A_u^{(0)} = 0 = e\Phi^2(-2\chi_{,u} + eA_u^{(0)}), \quad (78)$$

and another one for the massive field

$$\square A_\mu^{(m)} = J_\mu = e\Phi^2(-2\chi_{,\mu} + eA_\mu^{(m)}). \quad (79)$$

From the first equation we get $2\chi_{,u} = A_u^{(0)}$, which can be easily integrated leading to

$$\chi = \chi_0(\zeta, \bar{\zeta}) + \frac{e}{2} \int A^{(0)} du = \chi_0(\zeta, \bar{\zeta}) + \frac{e}{2i\omega} [A_u^{(0+)} - A_u^{(0-)}]. \quad (80)$$

Since the term $\chi_{,u}$ has been already eaten by the term $A_u^{(0)}$ we have got for the massive u-component the equations

$$\square A_u^{(m\pm)} = J_u^\pm = e^2\Phi^2 A_u^{(m\pm)}. \quad (81)$$

However, the mass term $e^2\Phi^2$ is not constant in our case, but is a function of ρ which fall off outside the string core. It complicates the regularization procedure since the solution of this equation will represent an interpolation between some solution with mass at the string core and the massless solution in far zone.

To solve this equation we use the ansatz

$$A_u^{(m\pm)} = e^{f_{\pm}(\rho)} A_u^{(0\pm)} \quad (82)$$

which will provide a similarity of the massive and massless components near the string core if the function $f_{\pm}(\rho)$ tends to zero by $\rho \rightarrow 0$.

Therefore, from (81) we obtain the equations

$$\square e^{f_{\pm}} A_u^{(0\pm)} = e^{\Phi^2} e^{f_{\pm}} A_u^{(0\pm)}. \quad (83)$$

It turns out that we can use a common function $f(\rho)$, or $f_+ = f_- = f(\rho^2) \equiv f(\zeta\bar{\zeta}/2)$.¹⁰

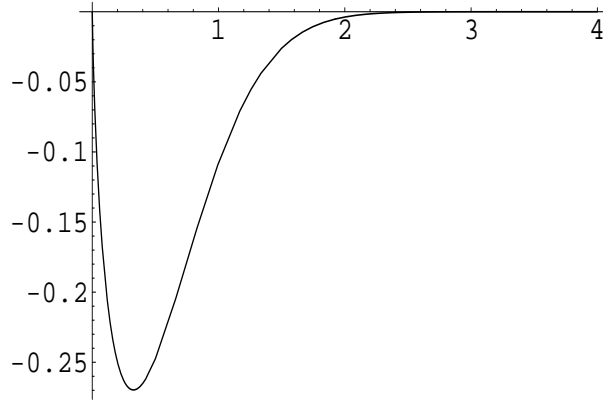


Figure 4: Regularizing dependence $f'(\rho)$ for the Gaussian distribution of condensate on the string.

Taking into account (78) we obtain from (83) the following differential equation

$$f'' + (f')^2 - f'/\rho = 8e^2\Phi^2. \quad (84)$$

¹⁰By using the coordinates $\zeta, \bar{\zeta}$ and the relations $\zeta\bar{\zeta} = 2\rho^2$, $\rho_{,\zeta} = \frac{\bar{\zeta}}{4\rho}$, $\rho_{,\zeta\bar{\zeta}} = \frac{1}{8\rho}$ computations are simplified.

Assuming for the shape of superconducting condensate on the string the distribution

$$\Phi(\rho) = ce^{-\rho^2}, \quad (85)$$

we obtain by numeric computations the following shape of the function $f'(\rho)$, which is shown on the fig.4.

The resulting regularized u-component takes the form

$$A_u = (1 - e^{f(\rho)})A_u^{(0)}, \quad (86)$$

and singularity cancels.

As it is seen from the fig.4, in the far zone the function $f(\rho) = \int_0^\rho f'(x)dx$ will tend to some negative constant, and as the result the factor $1 - e^{f(\rho)}$ will partially suppress the solution outside the string core too.

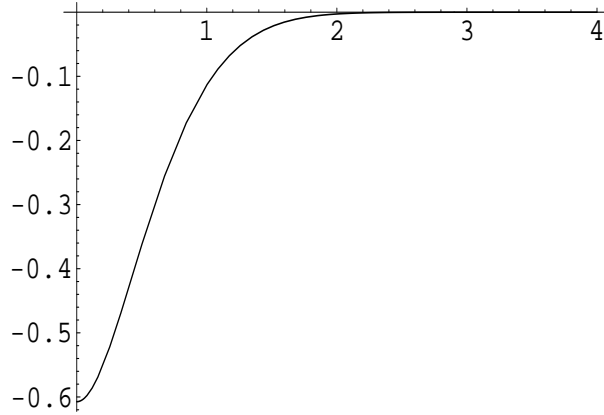


Figure 5: The dependence of massive components $A_\zeta^{(m)}$ and $A_{\bar{\zeta}}^{(m)}$ on the axial distance ρ .

For the other massive components the equations will retain the form (79), and from (80) one obtains

$$\chi_{,\zeta} = \chi_0(\zeta, \bar{\zeta})_{,\zeta} - \frac{e}{2i\omega \zeta} A_u^{(0+)}, \quad \chi_{,\bar{\zeta}} = \chi_0(\zeta, \bar{\zeta})_{,\bar{\zeta}} + \frac{e}{2i\omega \bar{\zeta}} A_u^{(0-)}. \quad (87)$$

Numerical computations allow us to obtain the behavior of the massive components $A_\zeta^{(m)}$ and $A_{\bar{\zeta}}^{(m)}$. The result of regularization is shown on the fig.5.

One sees that the used “compensatory” approach is really effective, and one can expect that it will also be effective for regularization of the Kerr ringlike singularity and by the formation of the Kerr’s disklike source.

8 Conclusion

The considered axial stringy system of the Kerr spinning particle gives a new view on the physical sense of the Dirac equation, and also maybe on the physical sense of the quantum wave function, divergences and regularization.

It should be also noted a striking similarity of this structure with the well known elements and methods of the signal transmission in the systems of radio engineering and radar systems. In fact, the chiral axial string resembles a typical system for the signal transmission containing a carrier frequency which is modulated by a signal - the wave function which is the carrier of information. The Kerr circular string also plays the role of a generator of the carrier frequency.

Basing on the principle that the fine description of a quantum system has to absorb maximally the known classical information on this system, one can conjecture that the above strikingly simple structure may have a relation to the structure of spinning particles. In this scheme the quantum wave function has a physical carrier. It should be mentioned that the considered topological coupling of the circular Kerr string and the axial stringy system reproduces the very old de Broglie’s “wave-pilot” idea on a singular carrier of the wave function which controls the dynamics of quantum particle.

We have already mentioned, that the axial string tension tends to zero for a free particle, but it has to be finite for the bounded states when the axial string can form the closed loops . One can also expect that for the closed loops an extra tension can appear caused by the magnetic flows which can concentrate on these strings [35], and it has to be a question for further investigations. The role of the axial strings in the mass renormalization, as well as the gravitational (pp-wave) nature of the axial strings contain a hint that such closed loops of the strings may be candidates for gravitons. One can also assume that vacuum itself may be formed from the strings of this type.

The existence of the axial stringy system in the Kerr spinning particle allows us to overcome the old contradiction between the very small experi-

mental value of the electron cross section and the very large Compton size of the Kerr source. One can conjecture now that the cross section of electron may be determined by the contact stringy interaction of the axial stringy system, acquiring also some corrections from the Compton region of the Kerr circular string.

It seems that the axial stringy system may be observable in the experiments with scattering of the soft laser beams on the crossing beams of polarized electrons and maybe on the crossing laser beams too, and this may be crucial for verification the reality of this axial stringy system.

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